There is quite a lot happening in this piece, *Work and Life Are One* by Rose Vickers.

Planar tiling

There are three shapes of tiles (rhombus, trapezoid, triangle) covering a flat board with no gaps. It's part of a tiling of a plane: the edges line up, and one can imagine how to extend the tiling in all directions forever. Ignoring colours for the moment, there are translational symmetries (if you imagine extending the board forever), reflections, and sixfold rotations.

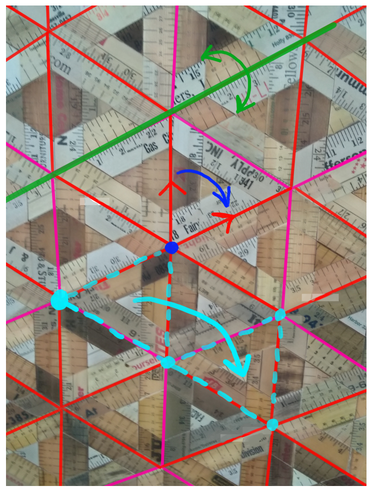

Figure 1. Some symmetries. There's a sixfold rotation about the dark blue point, a reflection across the green line, and a translation taking, say, one light blue triangle to the other.

One can imagine performing symmetries in sequence, stacking them together to get new symmetries. One can also "undo" symmetries (for example, you can rotate 60 degrees clockwise about the point in the centre; if you then rotate counterclockwise by 60 degrees, you're where you started). This means that the symmetries of the tiling form a mathematical object called a *group*. (A group is a collection of things, along with a rule for combining any two things in the group to get another thing in the group, subject to a few rules. For example, two clockwise 60–degree rotations combine to give a 120–degree rotation, etc.)

Remark. For what it's worth, the group is what a group theorist would call $\mathbb{Z}^2 \rtimes D_6$ – the "$\mathbb{Z}^2$" part is how we refer to symmetries that slide the whole picture around, moving every point, and the "D_6" part refers to symmetries, e.g. rotations, that fix some point. The "$\rtimes$" part is an operation for combining groups, reflecting how the latter symmetries interact with the former.

That's ignoring colour. The tiles are shaded in a variety of ways, and these have been carefully arranged to create an impression of depth.

Remark (The observer is reminded that the picture is actually finite)**.** My impression of the piece is that it's a "snapshot" of something infinite, because it's possible to imagine extending the tiling forever. I feel this very strongly, and the way I look at the piece depends on this feeling. The way this impression has been achieved is through the symmetry: enough repetitions of the pattern are shown that it's clear how to continue it beyond the confines of the frame.

Cubes

There are lots of hexagons. If you focus on just some of them, you can find collections that tile the plane snugly. But there are more; some of them overlap.

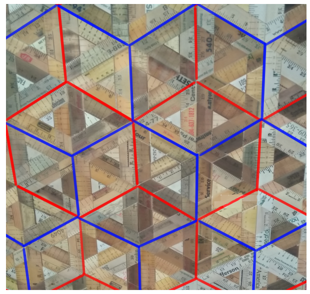

Figure 2. There is a tiling of the plane by red hexagons, and an overlapping tiling by blue ones. Each blue hexagon is cut into three pieces by the red lines in contains: it looks like the "back" of a cube. Each red hexagon is cut into pieces by blue lines; it's the "front" of a cube. Let your eye fill in the cubes, to get...

The impression that this creates — and this is very strongly suggested by the placement of tiles of different colours — is that we're looking at 3–dimensional cubes, some of which are in front of others. I feel that I'm looking at a piece of ordinary 3–dimensional Euclidean space, tiled by cubes (think of, say, a cubical building divided into cubical rooms of equal size). One big cube, divided into small ones, is made obvious in the centre of the piece by the use of shading.

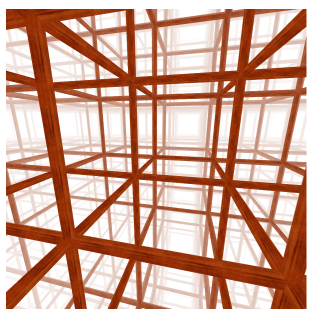

Figure 3. ...a tiling of 3-dimensional space by cubes. (Image from `https://en.wikipedia.org/wiki/Cubic_honeycomb`, in public domain.)

My eye is drawn to the smaller cubes that comprise the larger one. Although it's more ambiguous, the pattern persists among the lighter-coloured tiles near the edge of the piece.

Remark. In group-theory language, Rose has shown us a more or less literal picture of part of the *wallpaper group* $\mathbb{Z}^2 \rtimes D_6$ but cleverly conveyed the impression that we're looking at the much larger *space group* $\mathbb{Z}^3 \rtimes ((\mathbb{Z}/2\mathbb{Z})^3 \rtimes S_3)$. The $\mathbb{Z}^3$ part refers to the three dimensions worth of translation/sliding one can do, and the $((\mathbb{Z}/2\mathbb{Z})^3 \rtimes S_3)$ part refers to symmetries of individual cubes. (The notation isn't important, here, just the idea that symmetries of a tiling of space can be classified.)

Two-dimensional cross-sections of the cubical tiling of three-dimensional space are useful examples to think about in the type of maths I work on, so I'm very interested in whether Rose's piece also conveys this picture to other viewers, or whether it's a product of my preconceived notions.

Small scale

The 3–dimensionality is something you see on a large scale, looking at the whole piece, because it's conveyed by the changes in colour as your eye moves over the surface. (Can you imagine a different way to colour the tiles that suggests a different 3–dimensional picture?)

On a small scale, if you focus on just a few adjacent tiles, the colour changes are not so dramatic, and things flatten. You can still see individual cubes, but it feels more "optional", and the 2–dimensional pattern becomes more salient.

It's also when you zoom in that the phsysical technicalities of the piece reveal themselves. From far away, it's easy to believe you're looking at some sort of Platonic mathematical object (a tiling of a plane, or of 3–dimensional space), but up close, you can see how the tiles were formed and assembled (from rulers!). Wooden tiles require a lot more finesse and microadjustment than abstract mathematical tiles, and it's remarkable that the former — individually cut and assembled and adjusted by human hands — so readily call to mind the latter.

Process

One of the most interesting aspects of this project has been the discussions with Rose about the parallels between her working process as an artist and mine as a mathematician. In both cases, the process is about navigating constraints.

In maths, one is trying to figure out the necessary consequences of an elegant collection of assumptions. There are rules that constrain the "moves" one is allowed to make, and it's largely the constraints that give the results meaning and aesthetic weight: for example, it's not very interesting to prove a theorem that doesn't apply to many examples, or where the statement being proved is too close to the assumptions/inputs.

Similarly, Rose's work *evokes meaning* largely because there are very few types of tile (in this piece, just three, if you classify by shape) that are pieced together to create a pattern much larger than any of the individual tiles. This involves hard constraints: for example, there are various points in the piece where tiles are arranged around the point in cyclic fashion, e.g.:

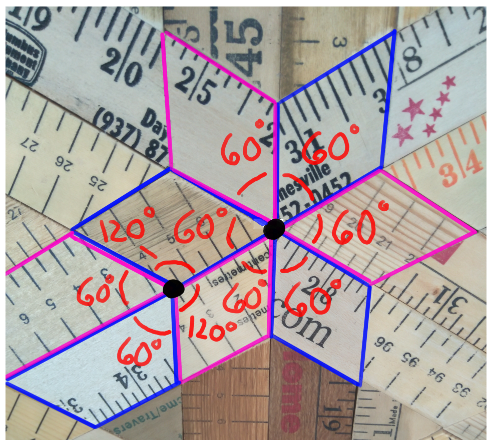

Figure 4. The angle constraint.

At each such point, each piece "contributes" one of its corners, and the angles formed at these corners must sum to 360°, because of the flatness of the board. Now, an artist who allowed themself as many types of tiles as they liked would always be able to satisfy this constraint in a "trivial" way, just by cutting whatever piece they needed to get out of whatever jam they were in. The result would likely have no interesting symmetry and would not achieve the same effect.

But by working within the constraints imposed by the flatness of the board, and just a few tile types, Rose has told us something concrete about the possibilities of tiling a plane in a highly symmetric way. Even more interestingly, she's done this in a way that guides us to similar thoughts in three dimensions, without literally showing us.